

Lecture 09/01/23 Linear Equations + Linear Inequalities

Defn: An equation is a mathematical statement that two expressions are equal.

Eg.  $3x - 2 = 7x - 1$ ,  $4x - 2 = 0$

Defn: A solution to an equation is any set of values that can replace the variables to produce a true statement.

~~The soln~~ A solution to  $4x - 2 = 0$  is  $x = \frac{1}{2}$   
Since

$$\begin{aligned} 4\left(\frac{1}{2}\right) - 2 & \stackrel{?}{=} 0 \\ 2 - 2 & \stackrel{?}{=} 0 \\ 0 & = 0 \cdot \checkmark \end{aligned}$$

Defn: A linear equation is an equation that can be written so that every term is a constant or a constant times one variable with no exponent.

Eg.  $2x + 4 = 13$   
 $2y + 4 = 2x - 1$   
 $x + y + z = 0$

Ex: Is  $x^2 + 3x + \frac{3}{x} = 0$  a linear equation? **No**

The steps to Solve a Linear Equation of one variable.

- 1) Simplify each side of equation separately.
  - a) Apply Dist. law to remove parentheses
  - b) Combine like terms.



2) Get all constants to one side and variable terms to the other

3) Divide both sides of equation by coefficient on variable term.

Ex: Solve the equation

$$-3(2x-9) = -10x + 29 + 2(2x-1)$$

Find all solutions:

$$1) \quad -6x + 27 = -10x + 29 + 4x - 2$$

$$-6x + 27 = -6x + 27$$

$$2) \quad 27 - 27 = -6x + 6x$$

$$0 = 0$$

Infinitely many sols.

Sometimes an equation will have multiple variables and we will want to solve for one of the variables in terms of the other.



$$P = 2L + 2W$$

i) Solve for L

$$P = 2L + 2W$$

$$P - 2W = 2L$$

$$\boxed{\frac{P - 2W}{2} = L}$$

ii) Solve for W

$$P = 2L + 2W$$

$$P - 2L = 2W$$

$$\boxed{\frac{P - 2L}{2} = W}$$

## Linear Inequalities

Defn: A ~~linear equation~~ inequality is an inequality is a statement like

$$a < b$$

a is less than b

$$a \leq b$$

a is less than or " $\leq$ " to b

$$a > b$$

a is greater than b

$$a \geq b$$

a is greater than or " $\geq$ " to b

Defn: A linear inequality is like a linear equation ~~equation~~ except instead of " $=$ " we have an inequality.

Eg  $2x + y > 2$  ,  $2x \geq 4$



We can solve linear inequalities much like we can solve linear equalities. Though we must be careful!

To solve a Linear Inequality

1) We may + or - Numbers to both sides of the inequality.

2) We may ~~multiply~~  $\times$  or  $\div$  ~~multiply~~ both sides of an inequality by a positive number

3) If we  $\times$  or  $\div$  both sides of an inequality by a negative number, we reverse the direction of the inequality symbol.

Ex: Solve ~~each inequality~~ ~~set~~ the inequality below. Graph its solution set and write its solution set in interval notation.

$$3 - 2x > 15$$

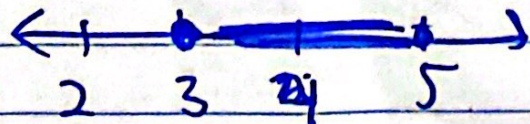
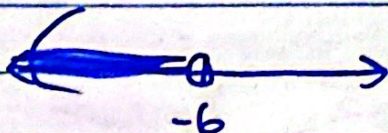
$$-2x > 12$$

$$x < -6$$

$$7 \leq -3x - 2 \leq 13$$

$$9 \leq -3x \leq 15$$

$$3 \geq x \geq 5$$



Ex: When is  $x - 3$



Positive

$$x - 3 > 0$$

$$\boxed{x > 3}$$

Negative

$$x - 3 < 0$$

$$\boxed{x < 3}$$

~~Zero~~

Zero

$$x - 3 = 0$$

$$\boxed{x = 3}$$